List of Topics – Basic Algebra Course

Groups

1. Definitions and examples of groups, subgroups, lateral classes, subgroup index, theorems of Lagrange, Euler and Fermat.
2. Normal subgroups, homomorphism, nucleus and image, isomorphism, fundamental theorem on homomorphism
3. Automorphism, conjugation, center, centralizer and normalizers.
4. Group actions on sets, orbits, fixed points, stabilizer, theorems of Cayley and Cauchy.
5. Symetric set Sn, conjugation classes of Sn and An, simplicity if An, for n >= 5, center and automorphism of Sn
6. Direct and semi-direct products
7. Solubility and nilpotent, derived series and central
8. Sylow's theorem and applications
9. Composition series, theorems of Jordan-Hölder and Schreier
10. Generators and relations, free sets

Ring Theory

1. Definition and examples of rings, ideals and morphism.
2. Chinese remainder theorem, Prime ideals, Maximal and characteristic.
3. Localization, fields of fractions of domains
4. Euclidean, principal and unique factorization domains
5. Polynomials, Lagrange interpolation, irreducibility, Gauss's lemma, symmetric polynomials, resultants and discriminants.
6. Modules and noetherian rings, Hilbert's basis theorem

Fields and Galois Theory

1. Field Extensions, Finite extensions, algebraic and normal extensions
2. Separable algebra
3. Filed authomorphism, fundamental theorem of Galois theory
4. Algebriac locks, Fundamental Theorem of Algebra
5. Finite fields, roots of unity, Constructability using ruler and geometric compass, roots of polynomials.

Linear Algebra

1. Free modules. Basis modules. Matrix and modules finitely generated over principal domains
2. Finitely generated abelian groups, structure and classification
3. Similarity to matrices over fields, rational and Jordan canonical forms, Matrix diagonalization, Cayley-Hamilton theorem, Jordan-Chevalley decomposition
4. Quadratic forms, Sylvester inertia theorem, definite positive and negative forms, orthogonal basis, hermitian forms, symmetric hermitian and normal matrix, congruence and orthogonal similarity.

References

Artin, E.	Geometric Algebra
Artin, E.	Galois Theory
Bourbaki, N.	Algèbre
Godement, R.	Cours d'algèbre
Herstein, I.N.	Topics in Algebra
Hungerford, T.W.	Algebra
Jacobson, N.	Basic Algebra I
Kaplansky, I.	Linear Algebra and Geometry
Lang, S.	Algebra
Rotman, J.	The Theory of Groups
van der Waerden, B.L.	Modern Algebra
Vargas, J.A.	Abstract Algebra
Zariski, O., Samuel, P.	Commutative Algebra I, II